The Tennis Formula: How it can be used in Professional Tennis

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Overview

- **Tennis scoring**
- **The tennis formula and its properties**
- **Other tennis-related formulas**
- **Applications of tennis formula**
- **Future work**

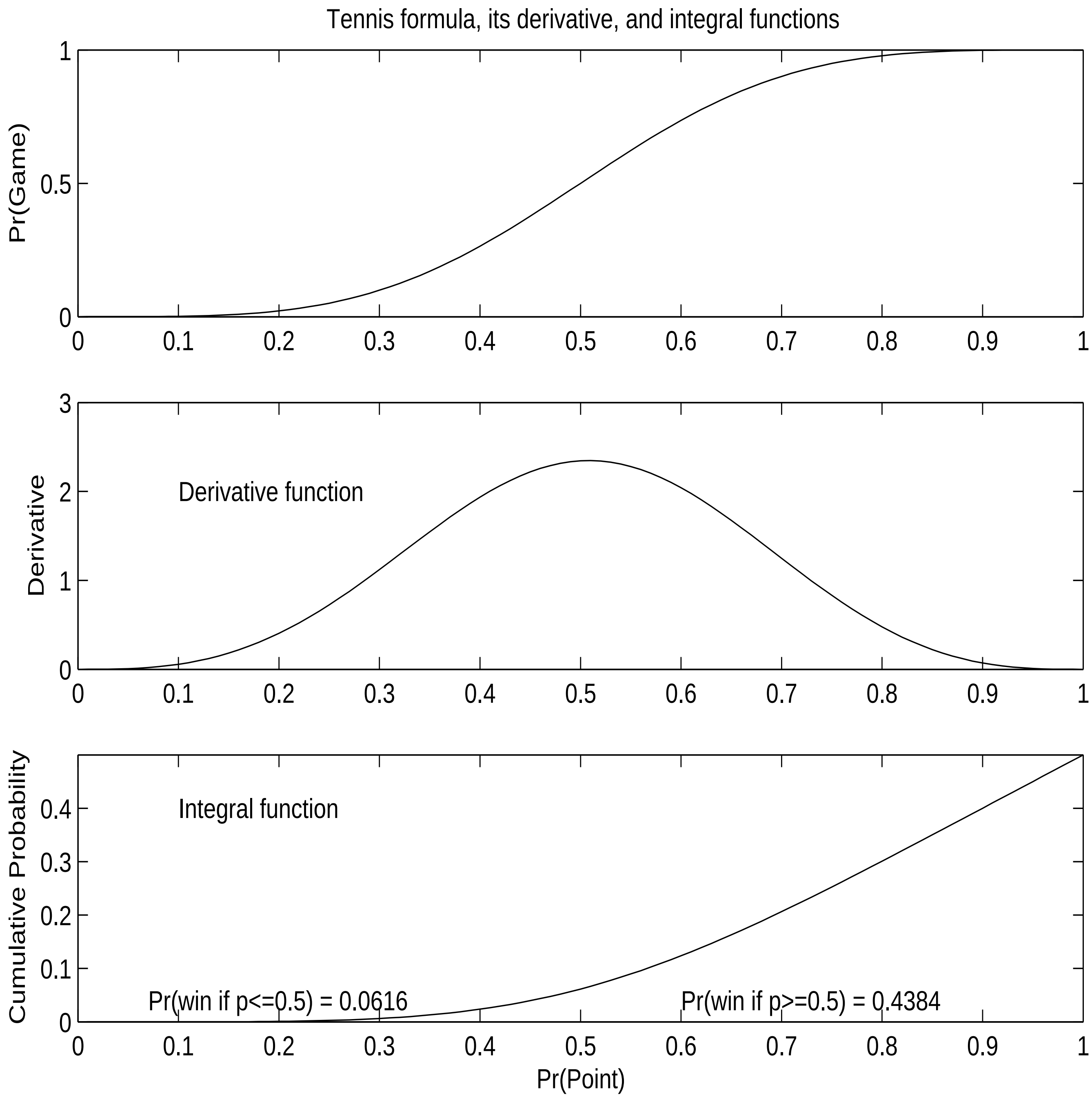
Scoring in tennis

- points: love, 15, 30, 40, deuce, advantage.
- games: love, 1, 2, 3, 4, 5, 6, (7).
- sets: love, 1, 2, (3).
- first to win four points or more by margin of two wins the game.
- first to win six games by margin of two or otherwise seven games wins the set (tiebreaker at six all).
- first to win two (or three) sets wins the match.

“Tennis Formula”

- **Let p denote the probability that a player wins a single point serving.**
- **Assume probability is fixed throughout game (match).**

$$\begin{aligned}\Pr(\text{Win game}) &= p^4 + 4p^4(1-p) + 10p^4(1-p)^2 \\ &\quad + 20p^3(1-p)^3 \cdot \frac{p^2}{1-2p(1-p)} \\ &= p^4\left(15 - 4p - \frac{10p^2}{1-2p(1-p)}\right)\end{aligned}$$


Tennis formula, its derivative, and integral functions
Pr(Game)
Derivative
Derivative function
Cumulative Probability
Integral function
Pr(win if p<=0.5) = 0.0616
Pr(win if p>=0.5) = 0.4384
Pr(Point)

Properties of tennis formula

- **Asymmetric - point of inflection at $p = 0.5$.**
- **Monotone increasing**
- **Derivative function reveals where improve performance is most beneficial.**

$$\frac{d\Pr(p)}{dp} = 20p^3\left(3 - p + \frac{5p^3 - 3p^2 + 4p^4}{(1 - 2p(1 - p))^2}\right)$$

- **Integral function gives probability of winning when serving probability selected at random.**

$$\int_0^p \Pr(x)dx = -\frac{2}{3}p^6 + 2p^5 - \frac{5}{4}p^4 - \frac{5}{6}p^3 + \frac{5}{4}p + \frac{5}{8}\log(1 - 2p(1 - p))$$

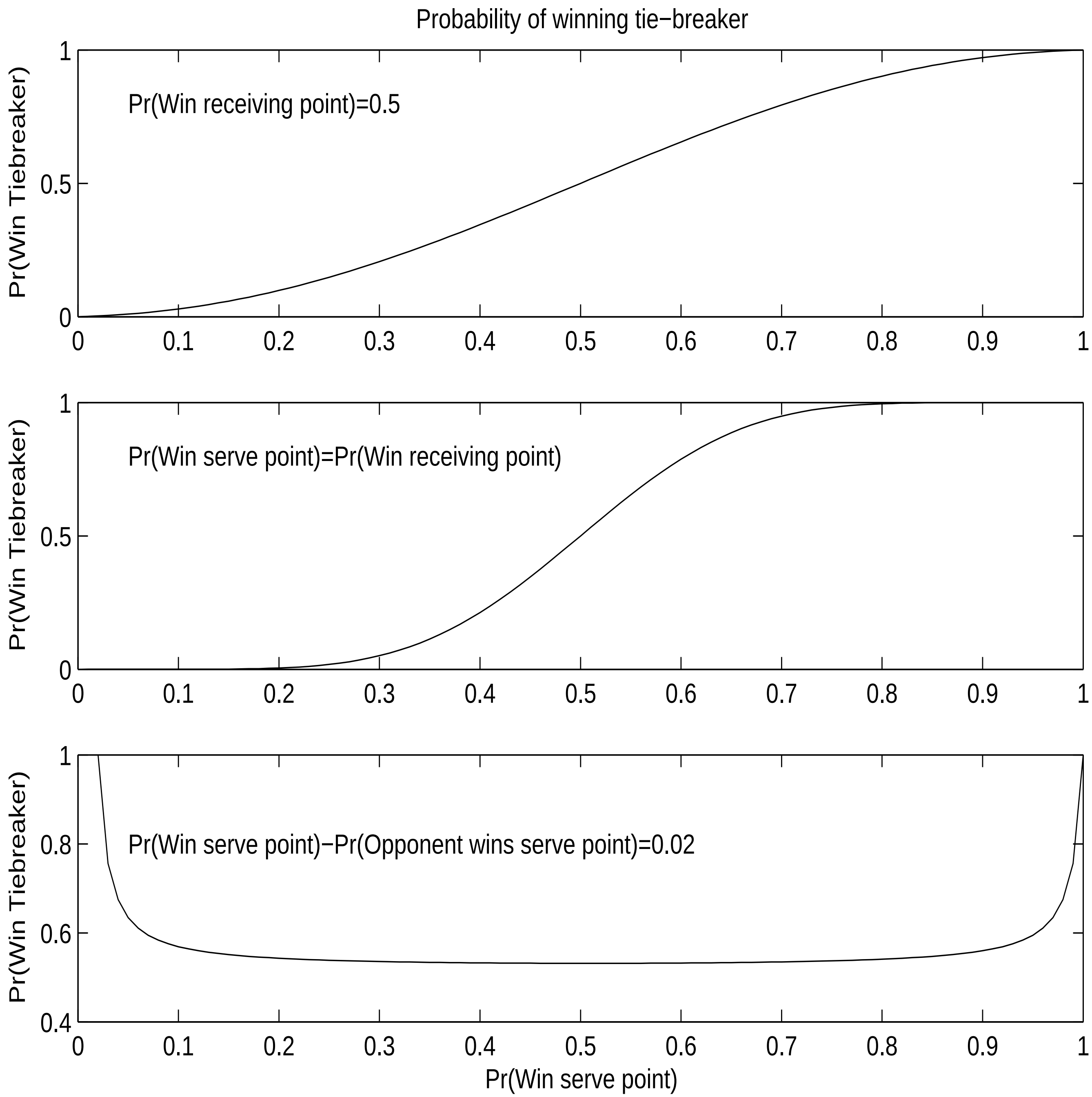
  - **Average over whole range $\int_0^1 \Pr(x)dx = 0.5$.**

Other probabilities

- Probability of winning:
  - tie-breaker.
  - set or match.
  - from a break down in final set.
- Derive similarly to the tennis formula; using tree diagram/dynamic programming approach.

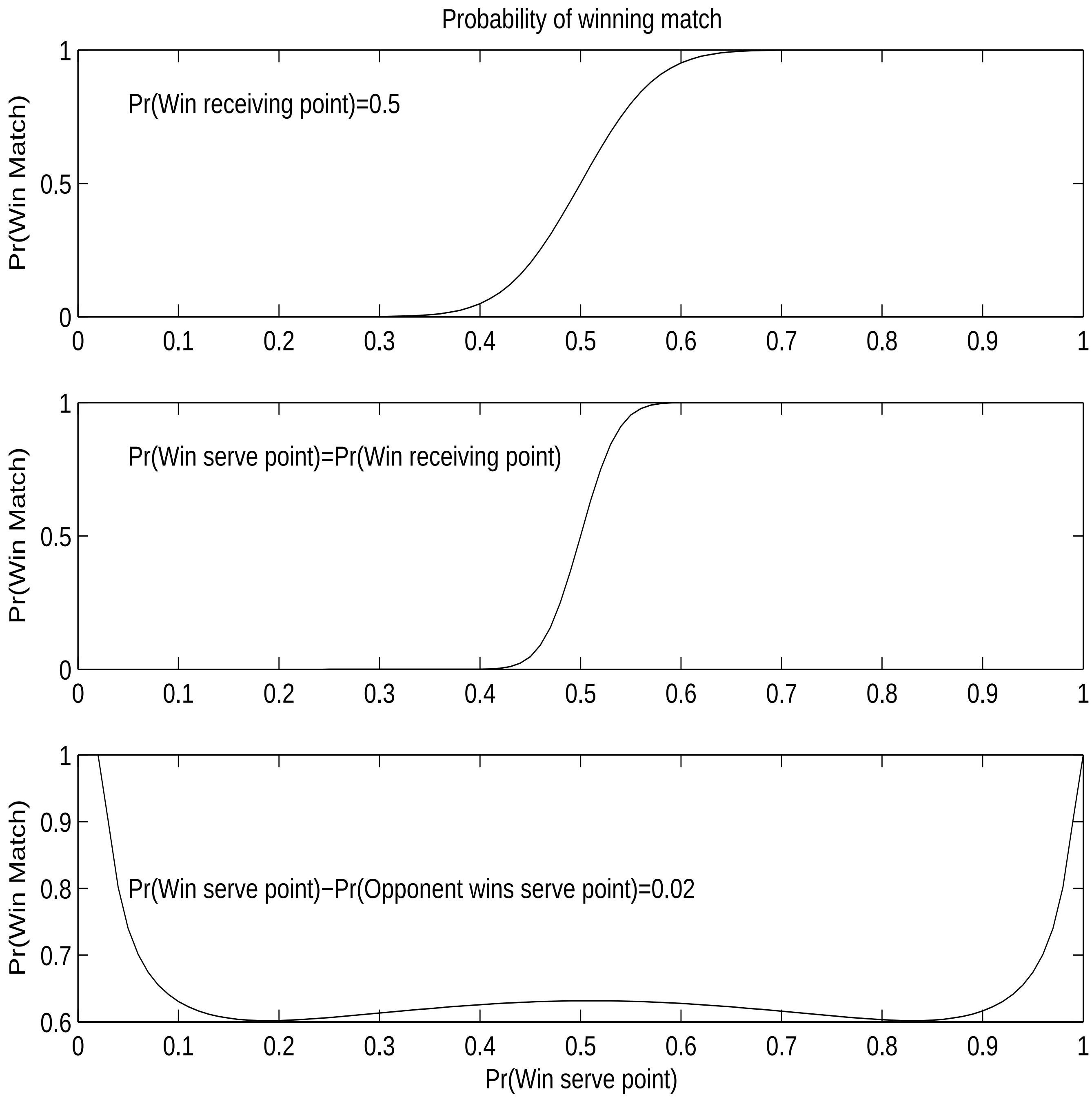
Probability of winning tiebreaker

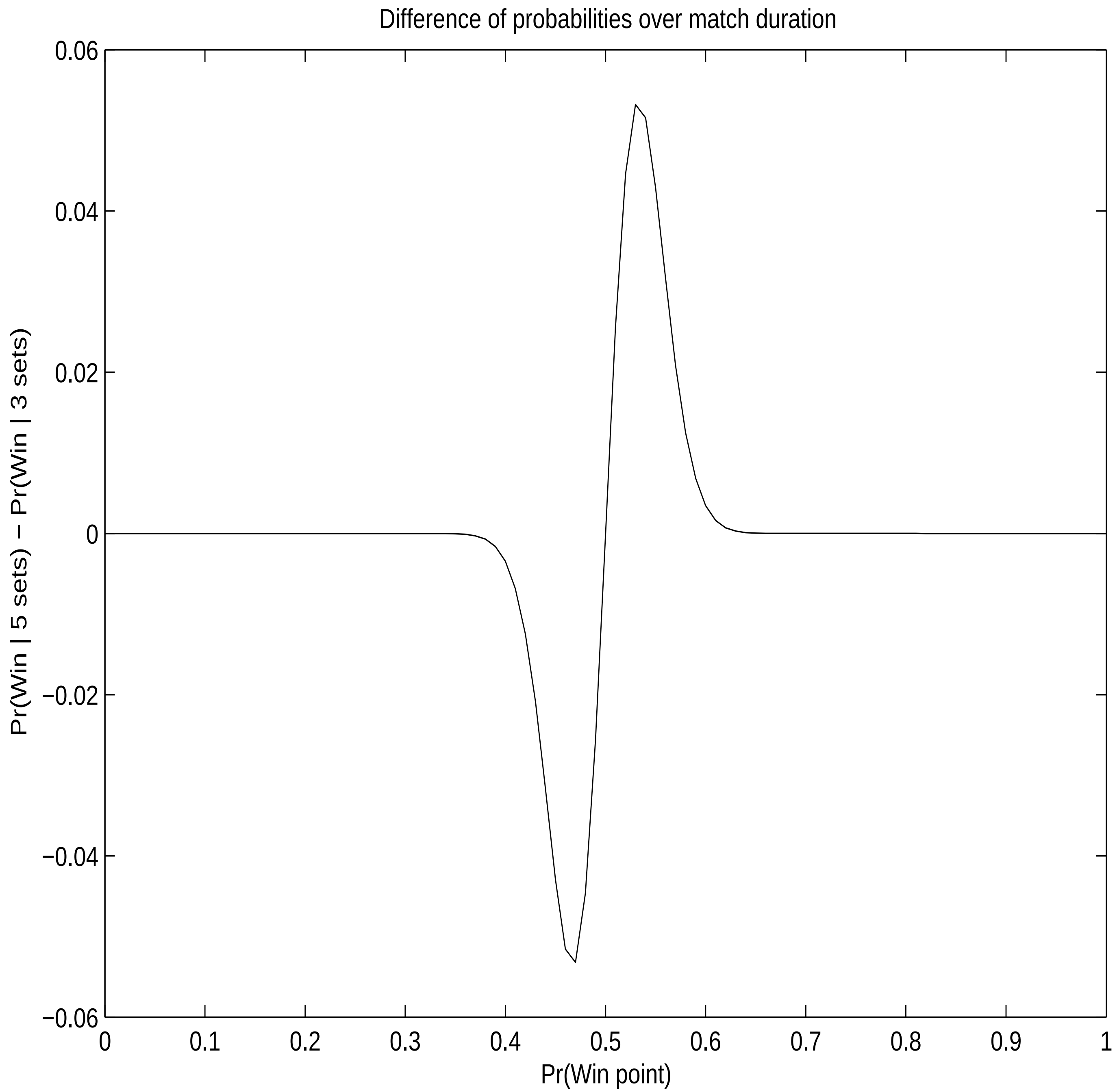
- **Tie-breaker is longer than a regular service game.**
  - **Involves both players serving, $q =$ opponents probability of winning point on serve.**
  - **When $q = 1 - p$ expect curve to be steeper than for the tennis formula.**


Probability of winning tie−breaker
Pr(Win receiving point)=0.5
Pr(Win serve point)=Pr(Win receiving point)
Pr(Win serve point)−Pr(Opponent wins serve point)=0.02
Pr(Win Tiebreaker)
Pr(Win serve point)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Probability of winning set and match

- **Functions of game and tie-breaker winning probabilities.**
  - **Thus, also of point-winning probabilities.**
- **Interested in how steeply odds favor better player.**

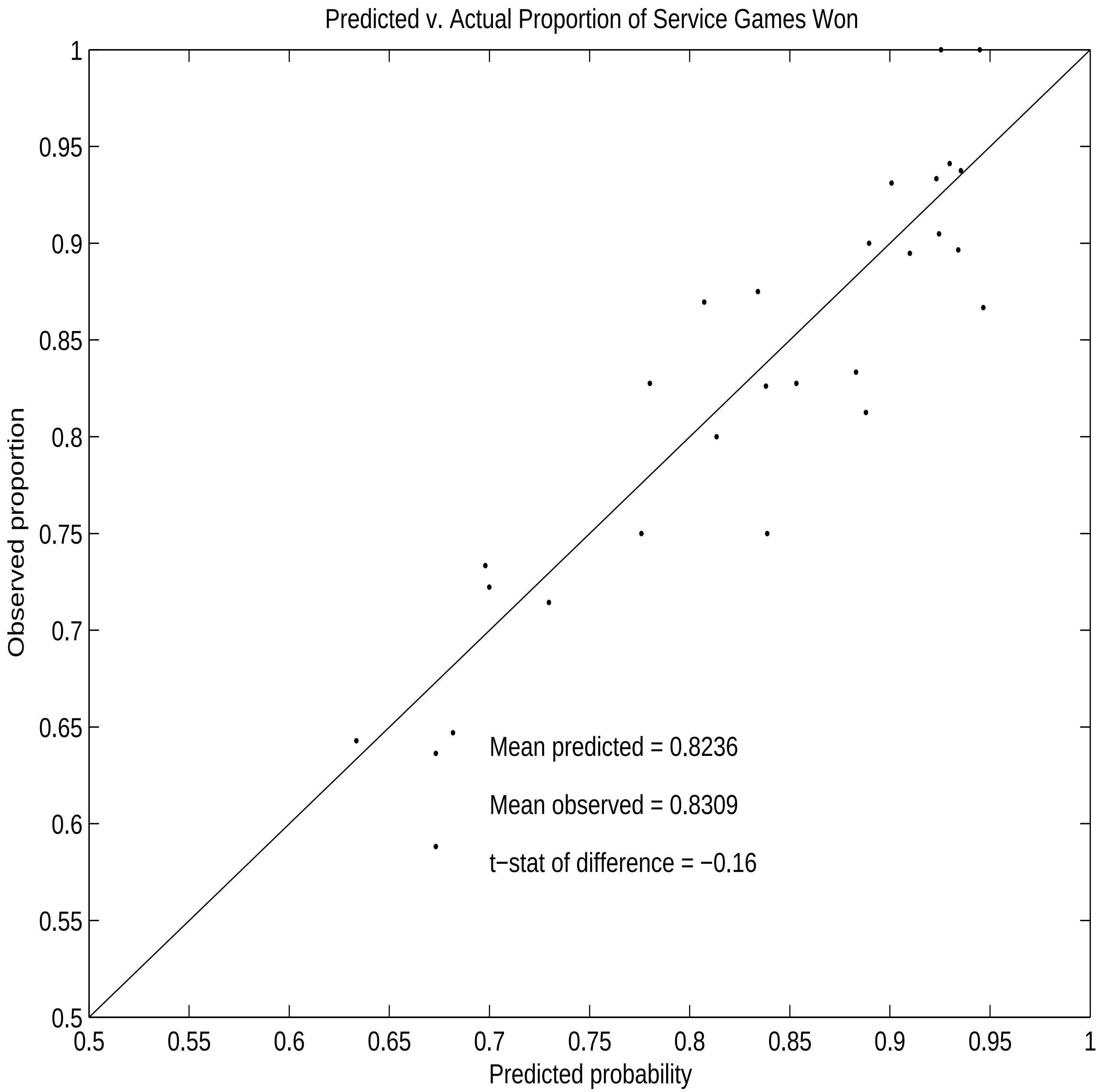

Probability of winning match
Pr(Win receiving point)=0.5
Pr(Win serve point)=Pr(Win receiving point)
Pr(Win serve point)−Pr(Opponent wins serve point)=0.02
Pr(Win Match)
Pr(Win Match)
Pr(Win Match)
Pr(Win serve point)



Difference of probabilities over match duration
Pr(Win | 5 sets) − Pr(Win | 3 sets)
Pr(Win point)
0.06
0.04
0.02
0
−0.02
−0.04
−0.06
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

Comparison of tennis formula to empirical data?

- **Formula's are based on assumptions:**
  - **Independence between points.**
  - **Homogeneous probabilities.**
- **Obtained data from Wimbledon 2007 (Mens singles).**
- **Compare empirical game winning percentages to predictions.**



Predicted v. Actual Proportion of Service Games Won
Observed proportion
Predicted probability
Mean predicted = 0.8236
Mean observed = 0.8309
t−stat of difference = −0.16

Lack of homogeneity of points across game

- **118 saves out of 208 break points, $p_{\text{save}} = 0.549$.**
- **2,101 out of 3,156 service points won at other stages of game, $p_{\text{other}} = 0.666$.**
- **P-value of difference ≈ 0.0053.**

Applications of tennis formula

- **By players to focus training efforts.**
- **By players to evaluate where to concentrate match preparation.**
- **By commentary teams to make broadcast more interesting.**
- **Useful in determining effect of a rule change.**

Training and match preparation

- Compute proportion of points one on serve and while receiving against all opponents.
- Evaluate corresponding probabilities of winning a match.
- Determine if more beneficial to improve serve or return game.
- Work on improving that aspect of game.
- Could extend this by averaging over types of opponents (left-handers, right-handers) to obtain more accuracy.
- Before playing a match analyze head-to-head data.

Example

- **Probability win service point = 0.65.**
- **Probability win receiving point = 0.37.**
- **Probability win 3 set match = 0.5985.**
- **Suppose focused training could improve serve probability by 1.1 percentage points or return by 1 percentage point. Where to focus effort?**
- **If improve service by 10%: Pr(match) = 0.6497.**
- **If improve return by 10%: Pr(match) = 0.6466.**
- **Better to improve serve!**

Making tennis commentary more interesting

- Report likelihood that each player wins match if:
  - Current point-winning percentage is maintained.
  - Players revert to historical winning proportions.
  - Probabilities became equal.
  - Stopped playing and tossed a coin.
- Calibrate statement "match is effectively over if player A breaks serve".

Chance of winning when break down in final set.

Situation	Scenario			
	$p = 0.62$ $q = 0.67$	$p = 0.67$ $q = 0.62$	$p = 0.645$ $q = 0.645$	$p = 0.5$ $q = 0.5$
4-5	0.1420	0.2165	0.1775	0.2500
3-5	0.0880	0.1451	0.1145	0.1250
2-5	0.0546	0.0972	0.0738	0.0625
3-4	0.2033	0.3038	0.2513	0.3125
2-4	0.1371	0.2217	0.1765	0.1875
1-4	0.0850	0.1486	0.1139	0.0938
2-3	0.2350	0.3526	0.2914	0.3438
1-3	0.1675	0.2716	0.2162	0.2266
0-3	0.1145	0.2006	0.1538	0.1367

Rule change

- In 1999 a change in the scoring of tennis was proposed.
- Replace deuce-advantage system with sudden death.
- At deuce the next point decides the game.
- Pete Sampras was against, Andre Agassi supported, the change.

New Tennis formula

- **Probability of winning game under new scoring system changes to:**

$$\mathrm{pr}(\mathrm{game} - \mathrm{new}) = p^4 + 4p^4(1-p) + 10p^4(1-p)^2 + 20p^4(1-p)^3$$

- **Compute change in probability of winning match.**

Sampras-Agassi Data (from 1999)

Statistic	Sampras	Agassi
Serving point	0.709	0.657
Return point	0.371	0.418
Pr(Win match - new)	0.8210	0.8092
Pr(Win match - old)	0.8331	0.8296
Net gain	-0.0121	-0.0205

Future work

- **More realistic models - allow probabilities to vary through stages of match.**
  - **At deuce, on break- or set-points, between sets.**
- **Use models to examine player performance at crucial stages of a match.**
  - **When to be most wary or optimistic against certain opponents.**