Using a tennis rating system to determine handicaps in amateur matches

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Introduction

• Handicap systems are used in sports to improve competitive balance
• Tennis does not have an official handicap system
• [1] propose a handicap system for tennis
  • The stronger player gives the weaker player h credits
  • The weaker player can use a credit at any time during the match
  • Whenever the weaker player uses one credit, she wins the point outright
  • The underlying mathematical model is a Markov Decision Process (MDP)
• [1] map server-specific point-win probabilities \((p_s, p_r)\) to handicap \(h\)

• However, determining point-win probabilities is challenging
• On the other hand, rating systems are used widely in tennis
• Question: Can we map a rating difference \(d\) to a handicap \(h\)?

High-level idea of our solution

• Final goal: Achieve a linear mapping of the form \(h = \gamma d\)
• Will use a sequence of mathematical models to achieve this mapping
• For amateurs, point-level data is sparse
• Hence, we start with the match scores and the ratings differences \(d\)

Model 1: Markov chain

• Goal: Map match-score to \(p\)
• Step 1: Map match-score to game-win probability \(q\)
  • Suppose match-score is 3-6, 4-0
  • We estimate \(q = (1 + 3)/(6 + 6 + 3 + 4) = 7/19\)
• Step 2: Map \(q\) to \(p\)
  • Use the Markov chain model for a single game of tennis
  • Can use the absorption probability equations to solve for \(p\) given \(q\)
• Validation: Validated on real data from 4131 ATP / WTA matches

Model 2: Bayesian model with logistic link

• Goal: Map \(p\) to \((p_s, p_r)\)
• Step 1: Map \(p\) to \(p_s\) using the following Bayesian model
  \[p_s \sim \mathcal{N}(p, \sigma)\]
  \[\frac{e^{\alpha p_s}}{1 + e^{\alpha p_s} + \beta p_s + \gamma p_s^2}\]
  Need to infer the three parameters \(\alpha, \beta,\) and \(\gamma\)
• Step 2: Infer \(p_s\) by calibrating to the match-win probability
• Estimation: Estimated using real data from 2465 WTA matches

Model 3: MDP model

Map \((p_s, p_r)\) to \(h\) using the MDP model proposed by [1]

Mapping ratings difference to handicap

• Goal: Map \(d\) to \(h\)
• We have already mapped match scores to handicap \(h\)
• Each match score has a corresponding ratings difference \(d\)
• Model: \(h = \gamma d\) fitted in a Bayesian fashion
  \[h \sim \mathcal{N}(\gamma d, \tau)\]
• Result: \(h \approx 11d\) (using data from 3686 amateur matches)

Heuristic

• Some players might not have a rating but know the expected set score
• Idea: Map set scores to handicaps

Implementation considerations

• Multiple ways a tennis organization can implement a handicap system
  • Way 1. Track data to estimate \(p\) and map to \(h\) using [1]
  • Way 2. Use the heuristic presented above
  • Way 3. Adopt a rating system and use the models presented here

Conclusion

• Developed a novel approach to map match scores to \(p\) to \((p_s, p_r)\)
• Rigorously mapped a tennis rating system to handicaps for amateurs
• Validated models on real data from thousands of matches
• Designed an easy to remember heuristic for handicaps
• Suggested ways to implement a handicap system in real life

References